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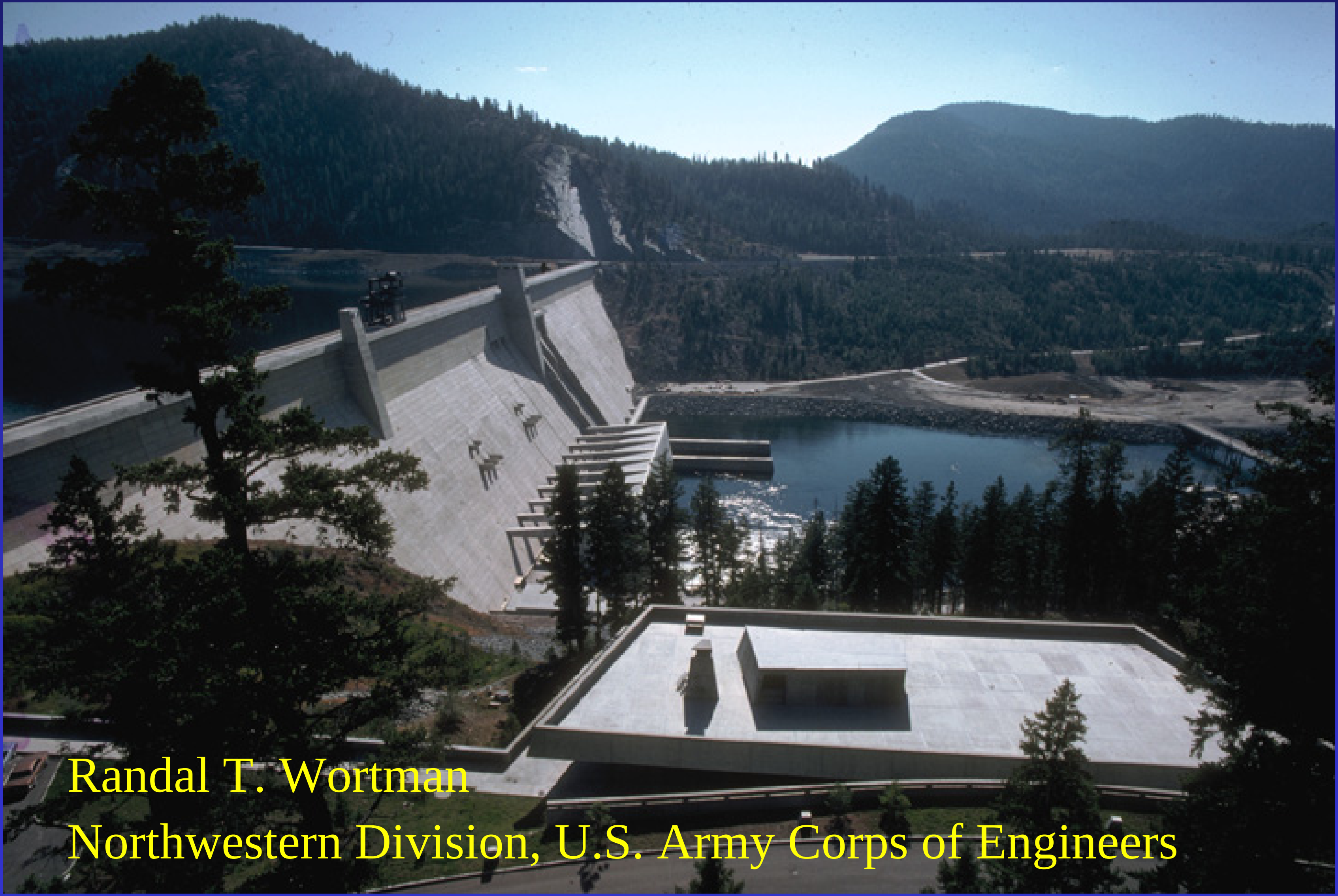
Regression Techniques in Water Supply Forecasting



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Where we are headed…

Observed Data

$X1_1$	$X2_1$	$X3_1$
$X1_2$	$X2_2$	$X3_2$
$X1_3$	$X2_3$	$X3_3$
$X1_4$	$X2_4$	$X3_4$
$X1_5$	$X2_5$	$X3_5$
$X1_6$	$X2_6$	$X3_6$
…	…	…
$X1_N$	$X2_N$	$X3_N$

Traditional Regression Model

$$Y = \beta_0 + \beta_1 * X1 + \beta_2 * X2 + \beta_3 * X3$$

Principal Components

$PC1_1$	$PC2_1$	$PC3_1$
$PC1_2$	$PC2_2$	$PC3_2$
$PC1_3$	$PC2_3$	$PC3_3$
$PC1_4$	$PC2_4$	$PC3_4$
$PC1_5$	$PC2_5$	$PC3_5$
$PC1_6$	$PC2_6$	$PC3_6$
…	…	…
$PC1_N$	$PC2_N$	$PC3_N$

Principal Component Regression Model

$$Y = \beta_0 + \beta_1 * PC1 + \beta_2 * PC2 + \beta_3 * PC3$$

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Standard Multiple-Variable Regression

- *Dependent variable is a seasonal inflow volume*
- *Predictor variables are pseudo-variables created from weighted combinations of similar stations (e.g. sum or average of three snow stations)*
- *Some models include variables not yet observed ("future variables") where the historic mean ("normal subsequent") is substituted in lieu of the variable being measured, e.g. using "May+June precip" in a 1-April forecast*

$$Y = \beta_0 + \beta_1 * X1 + \beta_2 * X2 + \beta_3 * X3 \ldots$$

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Regression Equation for Libby Water Supply Forecast

$$LibbyLocal_{APRAUG} = \beta_0 + \beta_1 * \sum_{Oct}^{Nov} LibbyLocal\ Runoff$$

$$+ \beta_2 * \sum_{Oct}^{Mar} (ELKB + FENB + FTIM + LRSM + BFEI + POLM)WinterPrecip$$

$$+ \beta_3 * (SUMB + NFRB + RMTM + KIMB + WSLM + 0.5 * MORB)Apr1\_Snow$$

$$+ \beta_4 * \sum_{Apr}^{Aug} w_i (FTIM + PTHI + KASB + WHFM)SpringPrecip$$

$$w_i \in \{1.0, 1.0, 0.8, 0.5, 0.2\}$$

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Concerns with Traditional Regression Models

- *Subjective station selection*
- *Subjective station weighting/aggregating*
- *Use of "normal subsequent" variable as a surrogate for a "future value" variable*
- *Variable selection and model selection based on model calibration statistics*
- *Predictor variables frequently highly intercorrelated*

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Principal Components

- *Creates surrogate variables (principal components) that are a weighted combination of the original variables.*
- *The principal components are fully independent of each other (zero intercorrelation)*
- *Most of the “variability” is loaded into the first one or two components.*
- *Eigenvalues reflect the proportion of the variability in original variables loaded into each component*
- *Note: PCs combine the information <u>within the predictor variables</u>, but “have no knowledge” of the dependent variable, the variable to be forecasted.*

Principal Components

Observed Data			Principal Components		
$X1_1$	$X2_1$	$X3_1$	$PC1_1$	$PC2_1$	$PC3_1$
$X1_2$	$X2_2$	$X3_2$	$PC1_2$	$PC2_2$	$PC3_2$
$X1_3$	$X2_3$	$X3_3$	$PC1_3$	$PC2_3$	$PC3_3$
$X1_4$	$X2_4$	$X3_4$	$PC1_4$	$PC2_4$	$PC3_4$
$X1_5$	$X2_5$	$X3_5$	$PC1_5$	$PC2_5$	$PC3_5$
$X1_6$	$X2_6$	$X3_6$	$PC1_6$	$PC2_6$	$PC3_6$
…	…	…	…	…	…
$X1_N$	$X2_N$	$X3_N$	$PC1_N$	$PC2_N$	$PC3_N$

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Principal Components Regression

- ***Methodology:***
  - P-variables * weighting matrix -> P principal components
  - Multivariable regression:
    - Use the P principal components as the predictor variables
    - Use the original WSF runoff variable as the dependent variable.

$$Y = \beta_0 + \beta_1 * PC1 + \beta_2 * PC2 + \beta_3 * PC3 \ldots$$

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Principal Components Regression

- ***Properties of the Principal Component Regression Model with all "P" Components ("p"= # of original variables)***
  - Component R-squared values
  - Component R-squared loading
  - Eigenvalue loading
- **7 original variables -> 7 principal components:**

PC Analysis	PC_1	PC_2	PC_3	PC_4	PC_5	PC_6	PC_7
Component R-Square	0.84144	0.00383	0.00040	0.00003	0.00495	0.00336	0.00068
Component R-Square %	98.5%	0.4%	0.0%	0.0%	0.6%	0.4%	0.1%
Cumulative R-Square	0.8414	0.8453	0.8457	0.8457	0.8506	0.8540	0.8547

Eigenvalues	4.41	0.80	0.74	0.55	0.36	0.09	0.04

Principal Components Regression

- ***Variable Selection (Which components do I keep?)***
  - **Component retention criteria:**
    - **Eigenvalue: provides the proportion of variability of X variables contained in each PC**
    - **significant R-squared: indicates the variability in the Y variable explained by this PC in a linear model, i.e. the usefulness of this PC in predicting the Y variable**
    - **significant *Beta:* reject this component when the regression coefficient is indistinguishable from zero.**
    - **sign of *Beta*: be wary of this component if the sign is negative (applies to water supply forecasting)**

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Dworshak Water Supply Forecast Example

- ***Predictor variables for Dworshak Water Supply Forecast:***
  - ***Climate variable: September SOI***
  - ***Precip variables: Monthly precip at Elk River for Dec & Jan***
  - ***Snow variables: 1-March SWE at 4 locations***
- ***We will be looking at the following:***
  - ***Correlation Analysis – Original Variables***
  - ***Standard Regression***
  - ***Stepwise Regression***
  - ***Correlation Analysis – Principal Components***
  - ***Principal Components Regression (All components)***
  - ***Principal Components Regression (Selected components)***

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Dworshak Water Supply Forecast Example

- ***Correlation Analysis – Original Variables (7)***

	Dwr_AprJul	*SOI_Sep*	*ELKDecPpt*	*ELKJanPpt*	*PierceMar*	*LostLkMar*	*HoodooMar*	*ElkButteMar*	*WYear*
Dwr_AprJul	1								
SOI_Sep	0.6309	1							
ELKDecPpt	0.5039	0.2665	1						
ELKJanPpt	0.5953	0.2872	0.2700	1					
PierceMar	0.7413	0.5857	0.3648	0.4760	1				
LostLkMar	**0.8367**	0.4140	0.4886	0.5128	0.6744	1			
HoodooMar	**0.8441**	0.4811	0.4939	0.5712	0.6276	**0.9300**	1		
ElkButteMar	**0.8664**	0.4798	0.4401	0.5696	**0.8010**	**0.9078**	**0.8495**	1	
WYear	-0.1818	-0.1211	-0.0647	-0.1697	-0.1183	-0.3165	-0.3073	-0.1903	1

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Dworshak Water Supply Forecast Example

- ***Standard Regression (fits all 7 predictor variables)***

Regression Statistics	
Multiple R	0.924488
R Square	0.854679
Adjusted R Square	0.826422
Standard Error	373.218
Observations	44

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	401.961	239.526	**1.678**	**0.102**
SOI_Sep	202.159	67.586	2.991	0.005
ELKDecPpt	33.610	27.035	**1.243**	**0.222**
ELKJanPpt	44.749	30.768	**1.454**	**0.155**
PierceMar	5.537	25.381	**0.218**	**0.829**
LostLkMar	6.504	13.613	**0.478**	**0.636**
HoodooMar	14.519	15.053	**0.964**	**0.341**
ElkButteMar	25.593	13.992	1.829	0.076

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Dworshak Water Supply Forecast Example

- ***Stepwise Regression (fits 3 predictor variables)***

Regression Statistics	
Multiple R	0.916011
R Square	0.839077
Adjusted R Square	0.827008
Standard Error	372.5874
Observations	44

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	508.027	222.764	2.281	0.028
SOI_Sep	201.763	59.391	3.397	0.002
HoodooMar	25.021	9.449	2.648	0.012
ElkButteMar	33.801	8.689	3.890	0.000

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Dworshak Water Supply Forecast Example

- ***Correlation Analysis – Principal Components***

	Dwr_AprJul	*PC7_1*	*PC7_2*	*PC7_3*	*PC7_4*	*PC7_5*	*PC7_6*	*PC7_7*	*WYear*
Dwr_AprJul	1								
PC7_1	**0.9173**	1							
PC7_2	-0.0619	0.0000	1						
PC7_3	-0.0199	0.0000	0.0000	1					
PC7_4	-0.0054	0.0000	0.0000	0.0000	1				
PC7_5	0.0703	0.0000	0.0000	0.0000	0.0000	1			
PC7_6	0.0579	0.0000	0.0000	0.0000	0.0000	0.0000	1		
PC7_7	0.0260	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1	
WYear	-0.1818	-0.2436	-0.0389	-0.0909	-0.0898	-0.2064	0.1400	0.1902	1

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Dworshak Water Supply Forecast Example

- ***Principal Components Regression (All 7 components)***

PC regression with all components is identical to standard regression with all variables =>

Regression Statistics	
Multiple R	0.924489
R Square	0.854679
Adjusted R Square	0.826422
Standard Error	373.218
Observations	44

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	2627.557	56.265	46.700	0.000
PC_1	391.149	27.092	14.438	0.000
PC_2	-61.982	63.613	**-0.974**	**0.336**
PC_3	-20.734	66.204	**-0.313**	**0.756**
PC_4	-6.565	76.988	**-0.085**	**0.933**
PC_5	104.582	94.466	**1.107**	**0.276**
PC_6	170.545	187.024	**0.912**	**0.368**
PC_7	110.065	268.496	**0.410**	**0.684**

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Dworshak Water Supply Forecast Example

- Select Components to Keep (or use Stepwise on PCs)
  - Component R-squared values
  - Component R-squared loading
  - Eigenvalue loading

PC Analysis	PC_1	PC_2	PC_3	PC_4	PC_5	PC_6	PC_7
Component R-Square	0.84144	0.00383	0.00040	0.00003	0.00495	0.00336	0.00068
Component R-Square %	98.5%	0.4%	0.0%	0.0%	0.6%	0.4%	0.1%
Cumulative R-Square	0.8414	0.8453	0.8457	0.8457	0.8506	0.8540	0.8547

Eigenvalues	4.41	0.80	0.74	0.55	0.36	0.09	0.04

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Dworshak Water Supply Forecast Example

- ***Principal Components Regression (1 component)***

Regression Statistics	
Multiple R	0.917301
R Square	0.841440
Adjusted R Square	0.837665
Standard Error	360.928
Observations	44

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	2627.561	54.412	48.290	0.000
PC7_1	391.149	26.200	14.929	0.000

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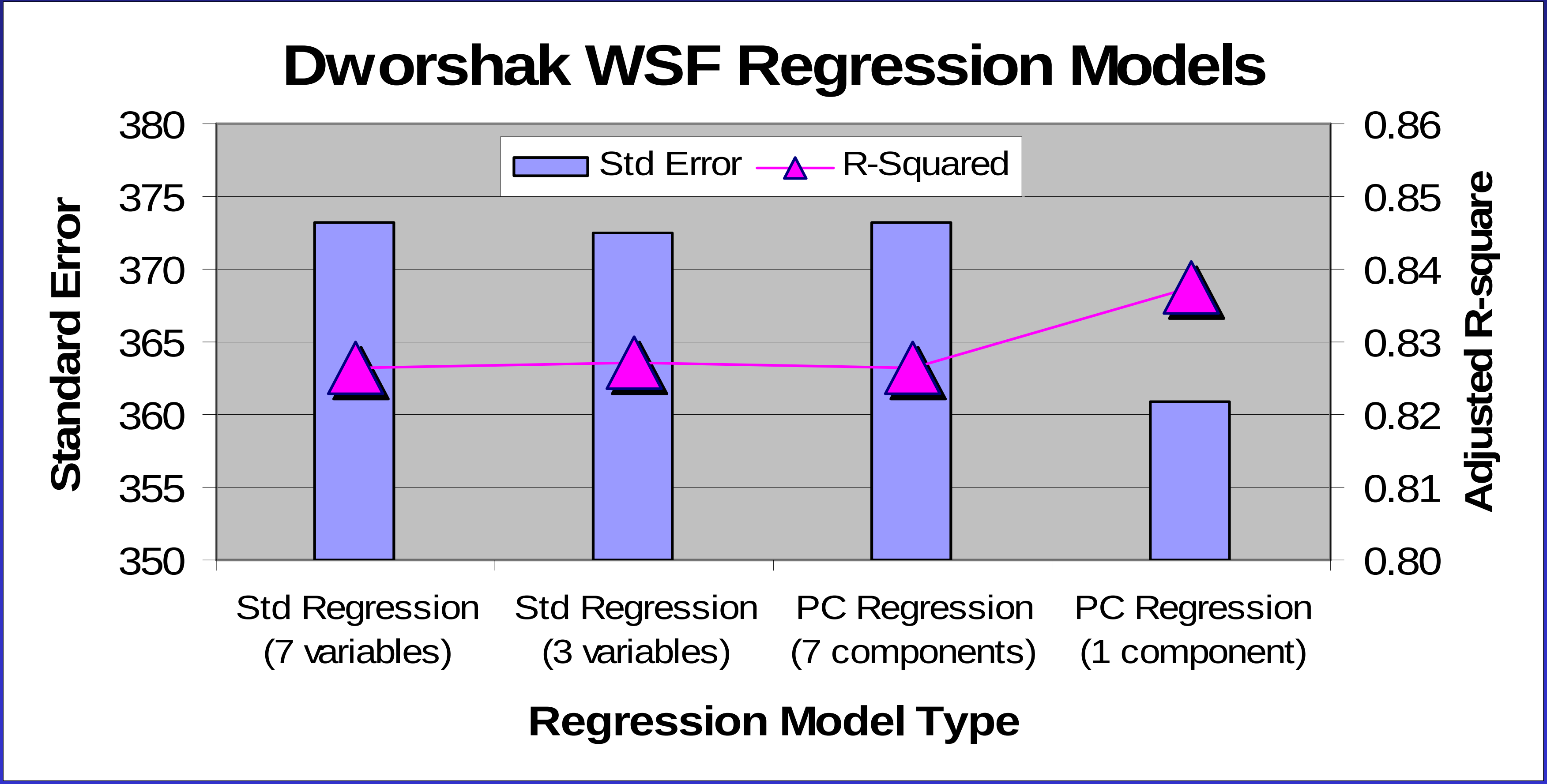
Model Comparison:
Standard Error & R-Squared statistics



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Model Comparison:
Cross-Validation Standard Error (CVSE)

- *Calibration vs. Validation statistics*
- *Split-Sample validation*
- *Leave-one-out validation*
- *Cross-validation standard error (“Jackknife” Std Err)*
- *CVSE supports model parsimony by including d-f adj.*
- *CVSE better indicator of how the model performs with data not used in the calibration, that it “hasn’t seen yet”*
- *CVSE = PRESS statistic adjusted for degrees-of-freedom*
- *CVSE can be directly calculated from either Projection matrix or “Hat” matrix (calculated by NRCS PCREG)*

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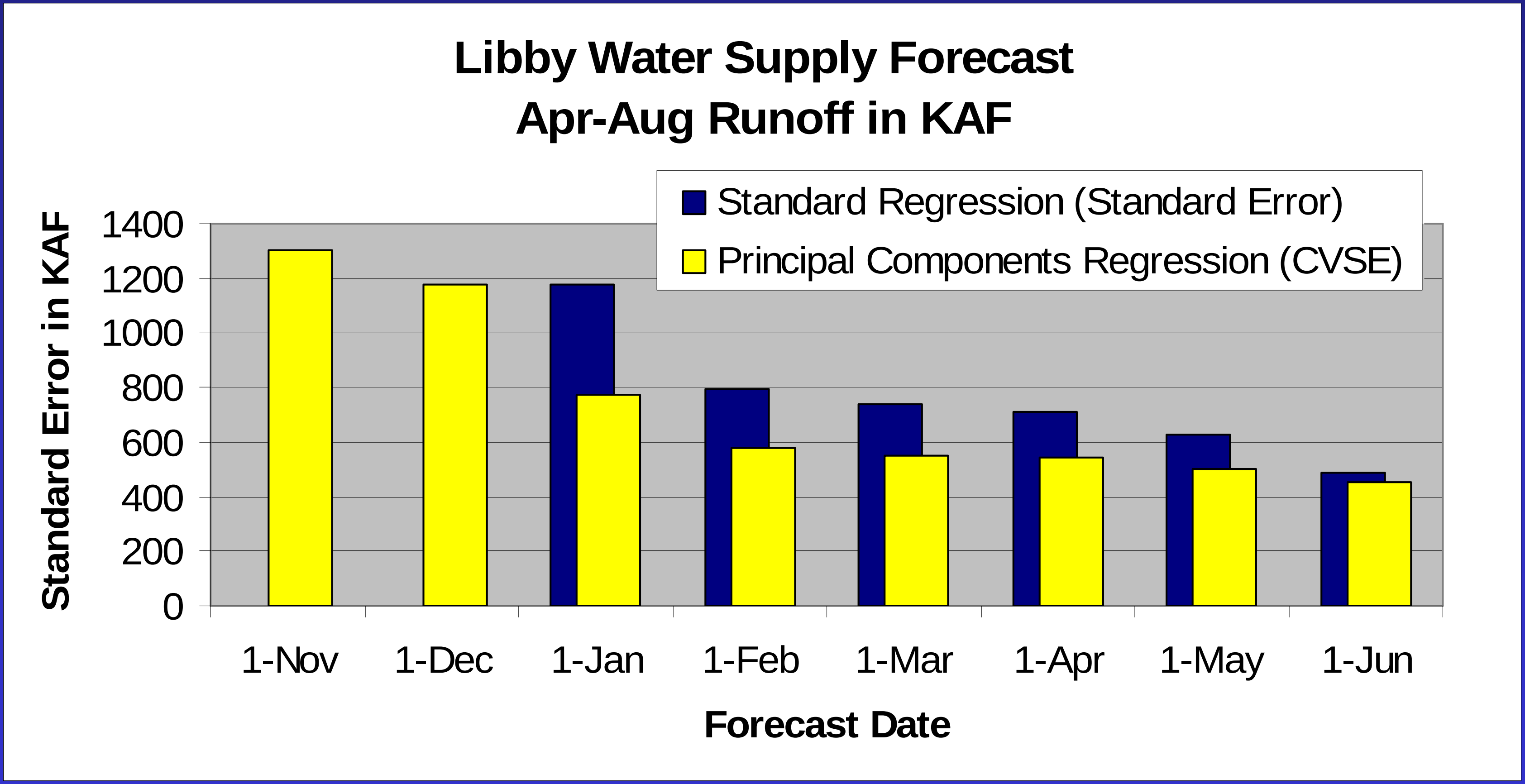
Libby Water Supply Forecast using Principal Components Regression

- *New equation for each month (similar pool of variables)*
- *Standard Error/CVSE plots*



Software for Regression Analysis

Capability	Software Package		
	Statgraphics	MS Excel	NRCS PCREG
Standard Regression	X	X	X
Stepwise Regression	X	X	
Adj R-square	X	X	
Standard Error	X	X	X
Principal Components	X	add-in	X
Eigen analysis	X	add-in	X
PrinComp Regression	X	indirectly	X
Cross Validation SE		indirectly	X
Optimal Search			X
Graphics	X	X	

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Endorsements

The following agencies use Principal Components regression in their Water Supply Forecasting procedures:

- *National Water and Climate Center, Natural Resources Conservation Service*
- *Northwestern Division, U.S. Army Corps of Engineers*
- *Northwest River Forecasting Center, National Oceanic and Atmospheric Administration*
- *Columbia River Treaty Operating Committee; Canadian and United States Entities*
- *Bonneville Power Administration, Dept of Energy*
- *BC Hydro*

Questions?